SAMPLE SIZE ESTIMATION FOR SOME STATISTICAL TESTS OF HYPOTHESES

Dr. Ibrahim Hassan Ibrahim
Faculty of Commerce, Tanta University

Key Words and Phrases : *Sample size, Power, Statistical inference, Normality assumption, Odds ratio .*

ABSTRACT

The problem of sample size determination has been one of the most important problem that faces researchers in different fields of scientific research because the final results of research depend on the chosen sample . This article provides some statistical tables that help the experimenters especially in the field of social sciences in determining sample size required for some common statistical tests of hypotheses . These tables are designed in a way that gives an estimation of sample size required for the given test with some known power and level of significance . The basic contribution of this article is the concentration on the process of estimating sample size that is required for statistical inferences . Also, saving time and efforts when determining the size of a sample from given tables without getting involved in tedious computations . One other main advantage of these tables is the increase in our ability of making comparisons between committing either type I or type II error and the cost of sampling unit .

1 . INTRODUCTION AND REVIEW OF LITERATURE

Theory of survey sampling has been discussed and presented by some well known statisticians such as Cochran (1977), Deming (1960), Hansen and Hurwitz (1986), Kish (1965), Scheaffer, Mendenhall and Ott (1986), and many others . The pivotal subject of all of these literature was the sample size determination required for estimation of some population parameters . It seems that the traditional way of presenting this theory of sampling is to divide sampling techniques into four major types of sampling : (i) the simple random sampling, (ii) the stratified random sampling, (iii) the systematic sampling, and (iv) the cluster sampling . Whereas, the main concern from using any one of these four sampling methods is to select sample size required for estimating population parameters such as mean, proportion, and population total value. Consequently, sample size determination required for testing statistical hypotheses was not a direct issue in any of these literature, the basic reason for this could be in the involvement of too many unknown parameters in the estimation process of sample size. However, Noethetr's approach (1987) was a different case from this traditional way of dealing with sampling theory. Noether discussed the problem of determining the number of observations required by some common nonparametric tests, so that these tests have minimum known power. The Noether's approach will be utilized in this article to manipulate the basic setup for constructing some statistical tables that help us in estimating sample size required for some common parametric statistical tests of hypotheses as well as some common nonparametric statistical tests of hypotheses. Needles to say the importance of such tables in practical applications.

From another point of view, there are some well known tables for estimating sample size that is needed in the field of auditing and accounting as well as in the field of quality control. Some of the well known statistical tables for auditors are provided by Brown, R.G. (1961) and some others in the same field of auditing and accounting are provided by Arkin, Herbert (1984). The main goal of the tables provided by either Arkin or by Brown is to give an estimate of the sample size required to achieve a given sample precision at a given confidence level, in other words, how close it is necessary for the auditor to estimate the population parameters. And of course, the main advantage of such tables from the point of view of the accountants and auditors is to avoid the awkward and time consuming work.

In the field of quality control, there are some known tables for sample size determination. One of the most important tables are the tables provided by the U.S.A Department of Defense known by the "Military Standard 105D", these tables are useful for acceptance sampling techniques that achieve some required goals for inspection and quality control. Montgomery, D.C. (1984) is a useful reference for further study in this subject.

The main goal of this article is to construct some statistical tables that provide an estimate of the sample size (n) required for some statistical tests of hypotheses with known power ($1 - \beta$) and level of significant (α). Hence, the basic difference between the tables suggested in this article and tables already known in literature is the concentration on statistical inference and the statistical power of the tests.

2. GENERAL SETUP

Since our goal is to estimate sample size required for testing some statistical hypotheses with known level of significant and a minimum power . Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from a density function $f(x ; \theta)$, where $\theta \in \Theta$, and let γ be a test statistic of the null hypothesis $H_0 : \theta \in \Theta_0$ versus the alternative hypothesis $H_1 : \theta \in \Theta_1 = \Theta - \Theta_0$. However, for simplicity, let us assume that we want to find the required sample size for testing that $H_0 : \theta \leq \Theta_0$ versus $H_1 : \theta > \Theta_0$. Assume also that γ is approximately normal with mean $\mu(\gamma)$ and standard deviation $\sigma(\gamma)$. This means that if the null hypothesis H_0 is true then $\gamma \sim N(\mu_0(\gamma), \sigma_0(\gamma))$. Hence, the decision rule is to reject H_0 if $\gamma > c$ with α - level of significant, where c is the critical region, i. e.,

$$\alpha = P(\text{Type I error} \mid H_0 \text{ True})$$
$$= P(\gamma > c \mid H_0)$$
$$= P(\gamma > \mu_0(\gamma) + Z_\alpha \sigma_0(\gamma) \mid H_0) \qquad \ldots (1)$$

where, Z_α is the upper α-level of significant, while the power function of that test is :

$$1 - \beta = P(\gamma > c \mid H_1)$$
$$= P(\gamma > \mu_0(\gamma) + Z_\alpha \sigma_0(\gamma) \mid H_1)$$

$$= P\left(\frac{\gamma - \mu(\gamma)}{\sigma(\gamma)} > \frac{\mu_0(\gamma) - \mu(\gamma) + Z_\alpha \sigma_0(\gamma)}{\sigma(\gamma)}\right)$$

$$= P\left(Z > \frac{\mu_0(\gamma) - \mu(\gamma)}{\sigma(\gamma)} + Z_\alpha \frac{\sigma_0(\gamma)}{\sigma(\gamma)}\right) \qquad \ldots (2)$$

Let $\eta = \sigma(\gamma)/\sigma_0(\gamma)$, which means that $\sigma(\gamma) = \eta\, \sigma_0(\gamma)$. Then the power function will be :

$$1 - \beta = P\left(Z > \frac{\mu_0(\gamma) - \mu(\gamma)}{\eta\, \sigma_0(\gamma)} + \frac{Z_\alpha}{\eta}\right) \qquad \ldots (3)$$

And because of the normality assumption we have :

$$\frac{\mu_0(\gamma) - \mu(\gamma)}{\eta\, \sigma_0(\gamma)} + \frac{Z_\alpha}{\eta} = -Z_\beta$$

$$\frac{\mu_0(\gamma) - \mu(\gamma)}{\eta\, \sigma_0(\gamma)} = -\left(\frac{Z_\alpha}{\eta} + Z_\beta\right)$$

multiplying both sides by $(-\eta)$, and squaring both sides we can get :

$$\lambda(\gamma) = \{ \frac{\mu(\gamma) - \mu_0(\gamma)}{\sigma_0(\gamma)} \}^2 = (Z_\alpha + \eta \ Z_\beta)^2 \qquad \dots (4)$$

where, $\lambda(\gamma)$ is the noncentrality parameter of the test .

In most cases η is unknown, but for the cases where $\sigma(\gamma)$ is close from $\sigma_0(\gamma)$, then under H_0 we can assume that η is close from unity, i. e., $\eta \approx 1$, hence

$$\lambda(\gamma) = \{ \frac{\mu(\gamma) - \mu_0(\gamma)}{\sigma_0(\gamma)} \}^2 = (Z_\alpha + Z_\beta)^2 \qquad \dots (5)$$

According to Noether (1987), we can find an approximate sample size if we solve equation number (5) for the number of observations for the given hypothesis . For example, if we let $x_1, x_2, x_3, \dots, x_n$ to be a random sample from normal population where, $X \sim N(\mu, \sigma^2)$, and if we are interested in testing the hypothesis $H_0 : \mu \leq \mu_0$ against the alternative $H_1 : \mu > \mu_0$, using the test statistic $\gamma = \bar{x}$. Then $\mu(\gamma) = \mu(\bar{x}) = \mu$, and $\sigma^2(\bar{x}) = \sigma^2 / n$, and, by substituting in equation (5) :

$$\lambda(\gamma) = \lambda(\bar{x}) = \{ \frac{\mu(\bar{x}) - \mu_0(\bar{x})}{\sigma_0(\bar{x})} \}^2 = (Z_\alpha + Z_\beta)^2$$

$$\{ \frac{\mu(\bar{x}) - \mu_0(\bar{x})}{\sigma / \sqrt{n}} \}^2 = (Z_\alpha + Z_\beta)^2$$

Then,

$$n = \frac{(Z_\alpha + Z_\beta)^2}{[(\mu - \mu_0)/\sigma]^2} \qquad \dots (6)$$

is the approximate sample size for the given test .

Equation (6) can also be obtained in a different way as follows :

$$\begin{aligned} \alpha &= P(\text{Type I error} \mid H_0 \text{ True}) = P(\text{Reject } H_0 \mid H_0 \text{ True}) \\ &= P(\bar{x} > c \mid H_0) \\ &= P(\frac{\bar{x} - \mu}{\sigma(\bar{x})} > \frac{c - \mu}{\sigma(\bar{x})} \mid H_0) \end{aligned}$$

$$= P\left(Z > \frac{c - \mu}{\sigma(\bar{x})} \mid H_0 \right)$$

$$\therefore\ \alpha = P(Z > Z_\alpha)$$

which means that $Z_\alpha = (c - \mu_0)/\sigma_0(\bar{x})$ ... (7)

$$\beta = P(\text{Type II error} \mid H_1 \text{ True}) = P(\text{Accept } H_0 \mid H_1 \text{ True})$$
$$= P(\bar{x} < c \mid H_1)$$
$$= P\left(\frac{\bar{x} - \mu}{\sigma(\bar{x})} < \frac{c - \mu}{\sigma(\bar{x})} \mid H_1 \right)$$
$$= P\left(Z < \frac{c - \mu}{\sigma(\bar{x})} \mid H_1 \right)$$

$$\therefore\ \beta = P(Z < -Z_\beta)$$

which also means that $-Z_\beta = (c - \mu) / \sigma(\bar{x})$ ... (8)

By subtracting equation (7) from equation (8) with the assumption that there is no difference between variability under both the null and the alternative hypothesis, where, $\sigma_0(\bar{x}) = \sigma(\bar{x}) = \sigma/\sqrt{n}$, then we have :

$$\left\{ \frac{\mu - \mu_0}{\sigma / \sqrt{n}} \right\} = -(Z_\alpha + Z_\beta)$$

Then, solving for n we get the normal approximation of sample size :

$$n = \frac{(Z_\alpha + Z_\beta)^2}{[(\mu - \mu_0)/\sigma]^2} \qquad \text{... (9)}$$

Equation (9) is identical to equation (6) for the right-tail hypothesis . Noting that the same equation is valid if we have a left-tail alternative, i. e., $H_1 : \mu < \mu_0$. However, for the two-tailed test $H_1 : \mu \neq \mu_0$, the same equation is applicable if every α is replaced by $\alpha/2$ and keep β less than 0.5 (Smith and William, 1977, p. 344) .

3. TABLES FOR ONE-SAMPLE TESTS

This section includes the process of constructing tables for estimating sample size required for testing : (i) population mean, (ii) population proportion, and (iii) population median . Tables for the one-sample tests, TABLE A-1, TABLE A-2, and TABLE A-3 are given in Appendix-A for the three previously mentioned tests receptively .

TABLE A-1 : Sample size required for testing population mean

In this case we are interested in testing $H_0 : \mu \leq \mu_0$ against the alternative $H_1 : \mu > \mu_0$. Equation (6) or (9) can be used for the one- tailed test, and for the two-tailed test $H_0 : \mu = \mu_0$, against $H_1 : \mu \neq \mu_0$, the same equation can be used if every α is replaced by $\alpha/2$ and keep β less than 0.5 as explained before . The main problem in equation (6) or (9) is the estimation of the absolute standardized difference Δ between the hypothetical and true mean, where $\Delta = | \mu - \mu_0 | /\sigma$. In real life applications we have to depend on some known information from experience or we can use pilot samples . TABLE A-1 of Appendix-A gives sample sizes required for testing population mean -one sided test- for different significant levels and powers for some positive values of $(\mu-\mu_0)/\sigma$ or for $0.01 \leq \Delta \leq 1.0$ since same results will be obtained if we have the same corresponding negative values .

TABLE A-2 : Sample size required for testing population proportion

Population proportion is an important parameter in survey research, where decisions about a specific population sometimes are made based on testing proportion resulted from a sample drawn from that population . For testing the null hypothesis $H_0 : \pi \leq \pi_0$, versus $H_1 : \pi > \pi_0$, where π is the population proportion of successes, and p is the sample proportion which is the appropriate test statistic where, p is approximately normal for large samples, $p \sim N(\pi, \pi(1-\pi)/ n)$. Then following the same steps previously explained in section 2 of this article, or by using equation (4) where,

$$\lambda(\gamma) = \lambda(p), \qquad \mu(\gamma) = \mu(p) = \pi, \qquad \mu_0(\gamma) = \mu_0(p) = \pi_0$$

and for $\sigma_0(\gamma) = \sigma_0(p) = \{\pi_0 (1 - \pi_0) / n \}^{1/2}$

then, $\eta = \sigma(p) / \sigma_0(p) = \{\pi (1 - \pi) /\pi_0 (1 - \pi_0) \}^{1/2}$.

Substituting in equation (4) we get the normal approximation of sample size based on the actual value of $\sigma^2(p)$:

$$\lambda(p) = \frac{\{\pi - \pi_0\}^2}{\pi_0 (1 - \pi_0) / n} = (Z_\alpha + \{\pi (1 - \pi) / \pi_0 (1 - \pi_0) \}^{1/2} Z_\beta)^2$$

By some manipulation and solving for n we can get the following :

$$n = \frac{(\{\pi_0 (1 - \pi_0)\}^{1/2} Z_\alpha + \{\pi(1 - \pi) \}^{1/2} Z_\beta)^2}{(\pi - \pi_0)^2} \qquad ...(10)$$

Similarly, if there is some believe that variability do not differ under both null and alternative hypotheses, then equation (10) can be simplified to equation (11) :

$$n = \frac{\pi_0 (1 - \pi_0) (Z_\alpha + Z_\beta)^2}{(\pi - \pi_0)^2} \qquad ...(11)$$

As it was discussed before there is usually a problem in determining the absolute value of $\Delta = | \pi - \pi_0 |$, because π is unknown . In this case we can depend on the past experience or some pilot samples . Noting that the same previous discussion is valid for left tail or two tail tests . TABLE A-2 of Appendix-A gives sample sizes required for testing population proportion -one sided test- for different significant levels and powers with given $\pi_0 = 0.1, 0.2, ..., 0.9$ and for some positive values of the difference $(\pi - \pi_0)$ or for $\Delta = 0.1, 0.2, ..., 0.9$ since same results will be obtained if we have the same corresponding negative values .

TABLE A-3 : Sample size required for testing population median

Let $x_1, x_2, x_3, ..., x_n$ be a random sample from a population with median M . If we are interested in testing $H_0 : M \le M_0$ versus $H_1 : M > M_0$ at a given α and power $(1 - \beta)$. The sign test can be used to achieve this goal . The test statistic is defined as $S = \# (x_i > M_0)$, where the distribution of S is binomial :

$S \sim Bin(n , p)$ with mean $\mu(S) = np$ and variance $\sigma^2(S) = np(1 - p)$, where $p = Pr(x_i - M_0 > 0)$. Under H_0 we have $\mu_0(S) = n/2$ and $\sigma_0^2(S) = n/4$. Then, substituting in equation (5) and solving for n we can get the normal approximation of sample size as follows :

$$\lambda(S) = \{ \frac{np - n/2}{\sqrt{n} / 2} \}^2 = (Z_\alpha + Z_\beta)^2$$

Then,

$$n = \frac{(Z_\alpha + Z_\beta)^2}{[2p - 1]^2} \quad \text{for } p \neq 1/2 \qquad ... (12)$$

However, if we have unequal variances, $\sigma_0^2(S) \neq \sigma^2(S)$, equation (4) should be used instead which will give the normal approximation of sample size based on the actual value of $\sigma^2(S)$, where

$$\eta = \sigma(S) / \sigma_0(S) = \{ np(1-p) / (n/4) \}^{1/2} = 2 \{ p(1-p) \}^{1/2}$$

$$\lambda(S) = \{ \frac{np - n/2}{\sqrt{n} / 2} \}^2 = (Z_\alpha + 2 \{ p(1-p) \}^{1/2} Z_\beta)^2$$

Then, $$n = \frac{(Z_\alpha + 2 \{ p(1-p) \}^{1/2} Z_\beta)^2}{[2p - 1]^2} \quad \text{for } p \neq 1/2 \qquad \ldots (13)$$

For determining the true probability p in either equation (12) or (13), Noether (1987) suggested the odds ratio (r) of number of observations greater than the median to number of observations less than the median where,

$r = \#(x_i - M_0 > 0) / \#(x_i - M_0 < 0) = p / (1 - p)$, which gives an estimate of $p = r / (1 + r)$, where r can be fixed by the experimenter. Equation (12) is used to construct TABLE A-3 of Appendix-A which gives some estimates of sample size required for testing population median for $(p > 1/2)$ and for different values of the α-level of significance and different β- levels . Noting that results are going to be the same if we considered the values of p or (1-p) as long as $p \neq 1/2$.

4. TABLES FOR TWO SAMPLE TESTS

The two sample tests considered in this article are : (i) testing the difference between two population means, (ii) testing the difference between two population distributions, and (iii) testing the significance of the population correlation coefficient . Appendix-B includes tables for the two-sample tests, TABLE B-1, TABLE B-2, and TABLE B-3 for the three previously mentioned tests respectively .

TABLE B-1 : Sample size required for testing the difference between two population means

If X_1 and X_2 are two independent random variables form normal distributions where, $X_1 \sim N (\mu_1 , \sigma_1^2)$ and $X_2 \sim N (\mu_2 , \sigma_2^2)$, then the difference between the population means can be tested using the difference between the two sample means $(\bar{x}_1 - \bar{x}_2)$. In this case we are interested in testing

$H_0 : \mu_1 \le \mu_2$ versus $H_1 : \mu_1 > \mu_2$. The test statistic for this test is defined as :

$\gamma = (\bar{x}_1 - \bar{x}_2)$,where $(\bar{x}_1 - \bar{x}_2) \sim N\ \{(\mu_1 - \mu_2), (\sigma_1^2 / n_1 + \sigma_2^2 / n_2)\}$. Consequently, $\mu(\gamma) = \mu_1 - \mu_2 = \mu^*$, and under H_0 we have $\mu_0(\gamma) = \mu_{01} - \mu_{02} = \mu_0^*$ and $\sigma_0^2(\gamma) = (\sigma_{01}^2 / n_1 + \sigma_{02}^2 / n_2)$. If the variances under both hypotheses are not different, then substituting in equation (5) :

$$\lambda(\bar{x}_1 - \bar{x}_2) = \left\{ \frac{\mu^* - \mu_0^*}{(\sigma_{01}^2 / n_1 + \sigma_{02}^2 / n_2)^{1/2}} \right\}^2 = (Z_\alpha + Z_\beta)^2 \qquad \text{... (14)}$$

If $n_1 = n_2$, and $\sigma_1 = \sigma_2 = \sigma$, then equation (14) can be written as follows :

$$\left\{ \frac{\mu^* - \mu_0^*}{(\sigma^2 / n + \sigma^2 / n)^{1/2}} \right\}^2 = (Z_\alpha + Z_\beta)^2$$

Solving for n will give :

$$n = \frac{2\,(Z_\alpha + Z_\beta)^2}{[(\mu^* - \mu_0^*)/\sigma]^2} \qquad \text{...(15)}$$

If $n_1 \ne n_2$, let $n_2 = k\,N$, where k is a constant, $0 < k < 1$, and $N = n_1 + n_2$, then $1/n_1 + 1/n_2 = (n_1 + n_2) / n_1 n_2 = 1 / n_1 k$. Equation (14) can be written as :

$$\left\{ \frac{\mu^* - \mu_0^*}{\sigma\,(1/n_1 k)^{1/2}} \right\}^2 = (Z_\alpha + Z_\beta)^2$$

And the normal approximation of sample size will be :

$$n_1 = \frac{(Z_\alpha + Z_\beta)^2}{k\ [(\mu^* - \mu_0^*)/\sigma]^2} \qquad \text{...(16)}$$

If $k = 1/2$, then equation (16) will be exactly as equation (15) where we have equal sample sizes .

Equation (16) is used to construct TABLE B-1 of Appendix-B . Values are given for the first sample for different level of significance and different power values, while for the second sample the following equation can be used $n_2 = n_1\ \{k/(1-k)\}$ for $k = 0.2, 0.4, 0.6, 0.8$.

TABLE B-2 : Sample size required for testing the difference between two population distributions

Let X and Y be stochastically independent random variables from distribution functions F(x) and F(y) respectively. If $x_1, x_2, x_3, \ldots, x_n$ and $y_1, y_2, y_3, \ldots, y_n$ are two random samples from these distributions, then we are interested in testing if the two samples have come from the same population or the values of Y tend to be larger than the values of X. Thus, the Mann-Whitney test statistic for testing $H_0 : F(x) = F(y)$ against $H_1 : F(x) > F(y)$ is defined as $U = \#(y_j > x_i)$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. If $p = Pr(y > x)$, then $\mu(U) = mnp$, and under the null hypothesis, $\mu_0(U) = mn/2$, and $\sigma_0^2(U) = mn(n+m+1)/12$. For large samples U is approximately normal, where $U \sim N\{\mu(U), \sigma^2(U)\}$. By the use of equation (5) we can estimate sample size as follows :

$$\frac{\{\mu(U) - \mu_0(U)\}^2}{\sigma_0^2(U)} = (Z_\alpha + Z_\beta)^2$$

$$\frac{\{mnp - mn/2\}^2}{mn(m+n+1)/12} = (Z_\alpha + Z_\beta)^2 \qquad \ldots(17)$$

Let $N = m + n$, and $m = kN$, then equation (17) can be written as :

$$\frac{kN^2(1-k)(p - 0.5)^2}{(N+1)/12} = (Z_\alpha + Z_\beta)^2 \qquad \ldots(18)$$

By solving equation (18) for N considering that for large samples (N +1) will be approximately N, then the total of the two samples will be :

$$N = \frac{(Z_\alpha + Z_\beta)^2}{12\,k\,(1-k)\,(p - 0.5)^2} \qquad \text{for } p \neq 1/2 \qquad \ldots(19)$$

Since $m = kN$ and $N = m + n$, then $n = N(k - 1)$, which will give the approximate normal estimation of each sample size by substituting in equation (19) as follows :

$$n = \frac{(Z_\alpha + Z_\beta)^2}{12\,k\,(p - 0.5)^2} \qquad \text{for } p \neq 1/2 \qquad \ldots(20)$$

$$m = \frac{(Z_\alpha + Z_\beta)^2}{12\,(1-k)\,(p-0.5)^2} \qquad \text{for } p \neq 1/2 \qquad \ldots(21)$$

If we have equal sample sizes, $n = m$ then, $N = 2n$ and $k = 1/2$ which will give the following estimation :

$$n = \frac{(Z_\alpha + Z_\beta)^2}{6\,(p-0.5)^2} \qquad \text{for } p \neq 1/2 \qquad \ldots(22)$$

Similarly, we can consider the odds ratio $r = p(y > x) / p(y < x) = p / (1 - p)$ and solving for p we can find $p = r / (1 + r)$. Equation (20) is used to find values in TABLE B-2 of Appendix-B which gives estimates of the first sample size required for testing the difference between two population distributions for values of p greater than 0.5 and for different values of the α-level of significance and different β- levels. While, for the second sample $m = n\,\{k/(1-k)\}$. Noting that results are going to be the same if we considered the values of p or (1-p) as long as $p \neq 0.5$.

TABLE B-3 : Sample size required for testing the correlation coefficient

Let (x_1,y_1), (x_2,y_2), (x_3,y_3), ..., (x_n,y_n) denote a random sample of size n from a bivariate normal distribution, where

$$X \sim N(\mu_1, \sigma_1^2) \quad \text{and} \quad Y \sim N(\mu_2, \sigma_2^2)$$

If we are interested in testing the population correlation ρ by the use of sample correlation r . Then the test statistic for testing the null hypothesis $H_0 : \rho \leq \rho_0$ against the alternative hypothesis $H_1 : \rho > \rho_0$, can be defined by using the Z-transformation (Yamane,1973,p.496) as Z_r, which is approximately normal, where $Z_r \sim N(\mu(Z_r), \sigma^2(Z_r))$ and the value of Z_r is defined as $Z_r = 0.5 \ln\{(1+r)/(1-r)\}$ with mean $\mu(Z_r) = Z_\rho = 0.5 \ln\{(1+\rho)/(1-\rho)\}$ and variance $\sigma^2(Z_r) = 1/(n-3)$. Similarly, if we use equation (5) we can find an estimate of the number of observations n as follows :

$$\lambda(Z_r) = \left\{ \frac{\mu(Z_r) - \mu_0(Z_r)}{\sigma_0(Z_r)} \right\}^2 = (Z_\alpha + Z_\beta)^2$$

$$\frac{[0.5 \ln\{(1+\rho)/(1-\rho)\} - 0.5 \ln\{(1+\rho_0)/(1-\rho_0)\}]^2}{1/(n-3)} = (Z_\alpha + Z_\beta)^2 \qquad \ldots(23)$$

$$(n-3)\,(Z_\rho - Z_{\rho_0})^2 = (Z_\alpha + Z_\beta)^2$$

where,

$$Z_{\rho} = 0.5 \ln\{(1+\rho)/(1-\rho)\} \quad \text{and} \quad Z_{\rho_0} = 0.5 \ln\{(1+\rho_0)/(1-\rho_0)\}$$

$$\therefore \quad n = \frac{(Z_{\alpha} + Z_{\beta})^2}{(Z_{\rho} - Z_{\rho_0})^2} + 3 \qquad \text{...(24)}$$

The previous equation shows that the estimated sample size depends upon the difference between the Z-transformation of the true and hypothetical correlation. TABLE B-3 of Appendix-B shows some estimates of sample sizes required for testing the correlation coefficient for some given values of $\Delta_z = | Z_{\rho} - Z_{\rho_0} |$ and for different values of the α-level of significance and different β- levels.

5. SUMMARY

The main concern of the traditional sampling theory is the estimation of sample size required for estimating population parameters. The approach of this article is some what different, where the main goal is to estimate sample size required for some common tests of hypotheses with given level of significance and known power. Tests considered in this article are the one-sample tests and the two-sample tests. For the one-sample tests three tables are provided to estimate sample size required for testing (i) population mean, (ii) population proportion, and (iii) population median. While for the two-sample tests, three other tables are constructed for estimating sample size required for testing (i) the difference between two means, (ii) the difference between two distributions, and (iii) the correlation coefficient.

One of the most important advantage of these tables, beside saving time and efforts, is the ability of making comparisons between different sample sizes with different power values and levels of significance. This comparisons is very important in scientific research, where decisions can be made about the risk of increasing type-I or type-II error and the cost of extra sampling units.

BIBLIOGRAPHY

(1) Arkin, Herbert, *Hand Book of Sampling for Auditing and Accounting*, 3rd, ed., McGraw-Hill : New York, 1984 .

(2) Cochran, W. G., *Sampling Techniques*, 3rd, ed., John Wiley : New York, 1977 .

(3) Conover, W. J., *Practical Nonparametric Statistics*, 2nd, ed., John Wiley : New York, 1980 .

(4) Deming, W. E., *Sampling Design in Business Research*, John Wiley : New York , 1960 .

(5) Hansen, M., Hurwitz, W., and Madow, W., *Sampling Survey Methods And Theory*, Vol. 1, John Wiley : New York, 1953 .

(6) Kish, L., *Survey Sampling*, John Wiley : New York, 1965 .

(7) Lehman, E. L., Dabrera, H., *Nonparametrics : Statistical Methods Based On Ranks*, Holden-Day, McGraw-Hill : San Francisco, 1975 .

(8) Montgomery, D. C., *Design and Analysis of Experiments*, 2nd, ed., John Wiley : New York, 1984 .

(9) Noether, Gottfried E., Sample Size Determination for Some Common Nonparametric Tests, *Journal of American Statistical Association*, Vol. 82, No. 398, pp. 645-647, 1987 .

(10) Noether, Gottfried E., *Elementary of Nonparametric Statistics*, John Wiley : New York, 1976 .

(11) Owen, D. B., *Handbook of Statistical Tables*, Addison-Wesley : Mass. , 1962 .

(12) Scheaffer, Mendenhall and Ott, *Elementary Survey Sampling*, 4th, ed., Duxbury Press : Boston, 1990 .

(13) Seymous Sudman, *Applied Sampling*, Academic Press : New York, 1976 .

(14) Smith, L., Williams, D., *Statistical Analysis for Business : A Conceptual Approach*, Wadsworth Publishing Inc. : Belmont, California, 1977 .

(15) Yamane, Taro, *Statistics An Introductory Analysis*, 3rd, ed., Harper & Row Publisher : New York, 1973 .

APPENDIXES

APPENDIX-A

TABLES FOR ONE-SAMPLE TESTS

TABLE A-1 : Sample size required for testing population mean .

TABLE A-2 : Sample size required for testing population proportion .

TABLE A-3 : Sample size required for testing population median .

APPENDIX-B

TABLES FOR TWO-SAMPLE TESTS

TABLE B-1 : Sample size required for testing the difference between two population means .

TABLE B-2 : Sample size required for testing the difference between two population distributions .

TABLE B-3 : Sample size required for testing the significance of correlation coefficient .

TABLE A-1

APPENDIX-A *Sample size required for testing population mean*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	22	16	13	9	11	9	5	7	4	2
0.99	22	16	13	9	11	9	5	7	4	2
0.98	23	16	14	9	11	9	6	7	4	2
0.97	23	17	14	10	12	9	6	7	4	2
0.96	23	17	14	10	12	9	6	7	4	2
0.95	24	17	14	10	12	9	6	7	4	2
0.94	25	18	15	10	12	10	6	7	4	2
0.93	25	18	15	10	13	10	6	8	4	2
0.92	26	19	15	11	13	10	6	8	5	2
0.91	26	19	16	11	13	10	6	8	5	2
0.90	27	19	16	11	13	11	7	8	5	2
0.89	27	20	16	11	14	11	7	8	5	2
0.88	28	20	17	12	14	11	7	8	5	2
0.87	29	21	17	12	14	11	7	9	5	2
0.86	29	21	18	12	15	12	7	9	5	2
0.85	30	22	18	12	15	12	7	9	5	3
0.84	31	22	18	13	15	12	8	9	5	3
0.83	31	23	19	13	16	12	8	10	6	3
0.82	32	23	19	13	16	13	8	10	6	3
0.81	33	24	20	14	16	13	8	10	6	3
0.80	34	25	20	14	17	13	8	10	6	3
0.79	35	25	21	14	17	14	9	11	6	3
0.78	36	26	21	15	18	14	9	11	6	3
0.77	37	27	22	15	18	14	9	11	6	3
0.76	37	27	23	16	19	15	9	11	7	3
0.75	38	28	23	16	19	15	10	12	7	3
0.74	40	29	24	16	20	16	10	12	7	3
0.73	41	30	24	17	20	16	10	12	7	3
0.72	42	30	25	17	21	17	10	13	7	4
0.71	43	31	26	18	21	17	11	13	8	4
0.70	44	32	27	18	22	17	11	13	8	4
0.69	45	33	27	19	23	18	11	14	8	4
0.68	47	34	28	19	23	19	12	14	8	4
0.67	48	35	29	20	24	19	12	15	9	4
0.66	50	36	30	21	25	20	12	15	9	4
0.65	51	37	31	21	26	20	13	16	9	4
0.64	53	39	32	22	26	21	13	16	9	4
0.63	55	40	33	23	27	22	14	17	10	5
0.62	56	41	34	23	28	22	14	17	10	5
0.61	58	42	35	24	29	23	14	18	10	5
0.60	60	44	36	25	30	24	15	18	11	5
0.59	62	45	37	26	31	25	15	19	11	5
0.58	64	47	39	27	32	25	16	20	11	5
0.57	67	49	40	28	33	25	17	20	12	6
0.56	69	50	42	29	35	27	17	21	12	6
0.55	72	52	43	30	36	28	18	22	13	6
0.54	74	54	45	31	37	29	18	23	13	6
0.53	77	56	46	32	39	30	19	23	14	6
0.52	80	58	48	33	40	32	20	24	14	7
0.51	83	61	50	35	42	33	21	25	15	7
0.50	87	63	52	36	43	34	22	26	15	7
0.49	90	66	54	38	45	36	22	27	16	8
0.48	94	68	57	39	47	37	23	29	17	8
0.47	98	71	59	41	49	39	24	30	17	8
0.46	102	75	62	43	51	40	25	31	18	9
0.45	107	78	64	44	53	42	27	32	19	9
0.44	112	81	67	47	56	44	28	34	20	9
0.43	117	85	70	49	59	46	29	36	21	10
0.42	123	89	74	51	61	49	30	37	22	10
0.41	129	94	77	54	64	51	32	39	23	11
0.40	135	99	81	56	68	54	34	41	24	11
0.39	142	104	86	59	71	56	35	43	25	12
0.38	150	109	90	62	75	59	37	45	26	13
0.37	158	115	95	66	79	63	39	48	28	13
0.36	167	122	100	69	84	66	42	51	30	14

Note : $\Delta = \mid \mu - \mu_0 \mid / \sigma$

TABLE A-1

APPENDIX-A *Sample size required for testing population mean* *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.35	177	129	106	74	88	70	44	54	31	15
0.34	187	136	113	78	94	74	47	57	33	16
0.33	199	145	120	83	99	79	49	60	35	17
0.32	211	154	127	88	106	84	53	64	37	18
0.31	225	164	135	94	113	89	56	68	40	19
0.30	241	175	145	100	120	95	60	73	43	20
0.29	257	188	155	107	129	102	64	78	45	22
0.28	276	201	166	115	138	109	69	84	49	23
0.27	297	216	179	124	148	117	74	90	52	25
0.26	320	233	193	133	160	127	80	97	57	27
0.25	346	252	208	144	173	137	86	105	61	29
0.24	376	274	226	156	188	149	93	114	66	32
0.23	409	298	246	170	205	162	102	124	72	34
0.22	447	326	269	186	224	177	111	136	79	38
0.21	491	358	295	204	245	194	122	149	87	41
0.20	541	394	325	225	271	214	134	164	96	45
0.19	600	437	361	249	300	237	149	182	106	50
0.18	668	487	402	278	334	264	166	203	118	56
0.17	749	546	450	312	374	296	186	227	132	63
0.16	846	616	509	352	423	335	210	257	149	71
0.15	962	701	579	400	481	381	239	292	170	81
0.14	1105	805	664	459	552	437	274	335	195	93
0.13	1281	933	770	533	640	507	318	389	226	108
0.12	1503	1095	904	625	752	595	374	456	266	126
0.11	1789	1303	1076	744	894	708	445	543	316	150
0.10	2165	1577	1302	901	1082	856	538	657	383	182
0.09	2673	1947	1607	1112	1336	1057	664	811	472	225
0.08	3383	2464	2034	1407	1691	1338	841	1027	598	284
0.07	4418	3219	2657	1838	2209	1748	1098	1341	781	371
0.06	6013	4381	3616	2502	3006	2379	1494	1825	1063	506
0.05	8659	6303	5207	3602	4329	3426	2152	2628	1531	728
0.04	13530	9857	8136	5628	6764	5353	3362	4106	2391	1137
0.03	24054	17524	14464	10006	12025	9516	5977	7300	4251	2022
0.02	54121	39428	32544	22514	27057	21411	13449	16425	9566	4550
0.01	216485	157712	130177	90054	108228	85644	53796	65700	38263	18198

TABLE A-2

Sample size required for testing population proportion

{ α = 1% , β = 1% }

π_0 \ Δ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	195	49	22	12	8	5	4	3	2
0.2	346	87	38	22	14	10	7	5	.
0.3	455	114	51	28	18	13	9	.	.
0.4	520	130	58	32	21	14	.	.	.
0.5	541	135	60	34	22	.	.	.	.
0.6	520	130	58	32	.	.	.	.	.
0.7	455	114	51	.	.	.	.	.	.
0.8	346	87	.	.	.	.	.	.	.
0.9	195	.	.	.	.	.	.	.	.

Note : $\Delta = |\pi - \pi_0|$

TABLE A-2

Sample size required for testing population proportion (Continued)

{ $\alpha = 1\%$, $\beta = 5\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	142	35	16	9	6	4	3	2	2
0.2	252	63	28	16	10	7	5	4	.
0.3	331	83	37	21	13	9	7	.	.
0.4	379	95	42	24	15	11	.	.	.
0.5	394	99	44	25	16	.	.	.	.
0.6	379	95	42	24	.	.	.	.	.
0.7	331	83	37	.	.	.	.	.	.
0.8	252	63	.	.	.	.	.	.	.
0.9	142	.	.	.	.	.	.	.	.

{ $\alpha = 1\%$, $\beta = 10\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	117	29	13	7	5	3	2	2	1
0.2	208	52	23	13	8	6	4	3	.
0.3	273	68	30	17	11	8	6	.	.
0.4	312	78	35	20	12	9	.	.	.
0.5	325	81	36	20	13	.	.	.	.
0.6	312	78	35	20	.	.	.	.	.
0.7	273	68	30	.	.	.	.	.	.
0.8	208	52	.	.	.	.	.	.	.
0.9	117	.	.	.	.	.	.	.	.

{ $\alpha = 1\%$, $\beta = 25\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	81	20	9	5	3	2	2	1	1
0.2	144	36	16	9	6	4	3	2	.
0.3	189	47	21	12	8	5	4	.	.
0.4	216	54	24	14	9	6	.	.	.
0.5	225	56	25	14	9	.	.	.	.
0.6	216	54	24	14	.	.	.	.	.
0.7	189	47	21	.	.	.	.	.	.
0.8	144	36	.	.	.	.	.	.	.
0.9	81	.	.	.	.	.	.	.	.

{ $\alpha = 5\%$, $\beta = 5\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	97	24	11	6	4	3	2	2	1
0.2	173	43	19	11	7	5	4	3	.
0.3	227	57	25	14	9	6	5	.	.
0.4	260	65	29	16	10	7	.	.	.
0.5	271	68	30	17	11	.	.	.	.
0.6	260	65	29	16	.	.	.	.	.
0.7	227	57	25	.	.	.	.	.	.
0.8	173	43	.	.	.	.	.	.	.
0.9	97	.	.	.	.	.	.	.	.

APPENDIX-A TABLE A-2

Sample size required for testing population proportion (Continued)

{ $\alpha = 5\%$, $\beta = 10\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	77	19	9	5	3	2	2	1	1
0.2	137	34	15	9	5	4	3	2	.
0.3	180	45	20	11	7	5	4	.	.
0.4	206	51	23	13	8	6	.	.	.
0.5	214	54	24	13	9	.	.	.	.
0.6	206	51	23	13	.	.	.	.	.
0.7	180	45	20	.	.	.	.	.	.
0.8	137	34	.	.	.	.	.	.	.
0.9	77	.	.	.	.	.	.	.	.

{ $\alpha = 5\%$, $\beta = 25\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	48	12	5	3	2	1	1	1	1
0.2	86	22	10	5	3	2	2	1	.
0.3	113	28	13	7	5	3	2	.	.
0.4	129	32	14	8	5	4	.	.	.
0.5	134	34	15	8	5	.	.	.	.
0.6	129	32	14	8	.	.	.	.	.
0.7	113	28	13	.	.	.	.	.	.
0.8	86	22	.	.	.	.	.	.	.
0.9	48	.	.	.	.	.	.	.	.

{ $\alpha = 10\%$, $\beta = 10\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	59	15	7	4	2	2	1	1	1
0.2	105	26	12	7	4	3	2	2	.
0.3	138	34	15	9	6	4	3	.	.
0.4	158	39	13	10	6	4	.	.	.
0.5	164	41	18	10	7	.	.	.	.
0.6	158	39	13	10	.	.	.	.	.
0.7	138	34	15	.	.	.	.	.	.
0.8	105	26	.	.	.	.	.	.	.
0.9	59	.	.	.	.	.	.	.	.

{ $\alpha = 10\%$, $\beta = 25\%$ }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	34	9	4	2	1	1	1	1	1
0.2	61	15	7	4	2	2	1	1	.
0.3	80	20	9	5	3	2	2	.	.
0.4	92	23	10	6	4	3	.	.	.
0.5	96	24	11	6	4	.	.	.	.
0.6	92	23	10	6	.	.	.	.	.
0.7	80	20	9	.	.	.	.	.	.
0.8	61	15	.	.	.	.	.	.	.
0.9	34	.	.	.	.	.	.	.	.

TABLE A-2

Sample size required for testing population proportion (Continued)

{ α = 25% , β =25% }

$\pi_0 \backslash \Delta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	16	4	2	1	1	1	1	1	1
0.2	29	7	3	2	1	1	1	1	.
0.3	38	10	4	2	2	1	1	.	.
0.4	44	11	5	3	2	1	.	.	.
0.5	45	11	5	3	2	.	.	.	.
0.6	44	11	5	3	.	.	.	.	.
0.7	38	10	4	.	.	.	.	.	.
0.8	29	7	.	.	.	.	.	.	.
0.9	16	.	.	.	.	.	.	.	.

TABLE A-3

Sample size required for testing population median

P	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	22	16	13	9	11	9	5	7	4	2
0.99	23	16	14	9	11	9	6	7	4	2
0.98	23	17	14	10	12	9	6	7	4	2
0.97	25	18	15	10	12	10	6	7	4	2
0.96	26	19	15	11	13	10	6	8	5	2
0.95	27	19	16	11	13	11	7	8	5	2
0.94	28	20	17	12	14	11	7	8	5	2
0.93	29	21	18	12	15	12	7	9	5	2
0.92	31	22	18	13	15	12	8	9	5	3
0.91	32	23	19	13	16	13	8	10	6	3
0.90	34	25	20	14	17	13	8	10	6	3
0.89	36	26	21	15	18	14	9	11	6	3
0.88	37	27	23	16	19	15	9	11	7	3
0.87	40	29	24	16	20	16	10	12	7	3
0.86	42	30	25	17	21	17	10	13	7	4
0.85	44	32	27	18	22	17	11	13	8	4
0.84	47	34	28	19	23	19	12	14	8	4
0.83	50	36	30	21	25	20	12	15	9	4
0.82	53	39	32	22	26	21	13	16	9	4
0.81	56	41	34	23	28	22	14	17	10	5
0.80	60	44	36	25	30	24	15	18	11	5
0.79	64	47	39	27	32	25	16	20	11	5
0.78	69	50	42	29	35	27	17	21	12	6
0.77	74	54	45	31	37	29	18	23	13	6
0.76	80	58	48	33	40	32	20	24	14	7
0.75	87	63	52	36	43	34	22	26	15	7
0.74	94	68	57	39	47	37	23	29	17	8
0.73	102	75	62	43	51	40	25	31	18	9
0.72	112	81	67	47	56	44	28	34	20	9
0.71	123	89	74	51	61	49	30	37	22	10
0.70	135	99	81	56	68	54	34	41	24	11
0.69	150	109	90	62	75	59	37	45	26	13
0.68	167	122	100	69	84	66	42	51	30	14
0.67	187	136	113	78	94	74	47	57	33	16
0.66	211	154	127	83	106	84	53	64	37	18
0.65	241	175	145	100	120	95	60	73	43	20
0.64	276	201	166	115	138	109	69	84	49	23
0.63	320	233	193	133	160	127	80	97	57	27
0.62	376	274	226	156	188	149	93	114	66	32
0.61	447	326	269	185	224	177	111	136	79	38
0.60	541	394	325	225	271	214	134	164	96	45
0.59	668	487	402	278	334	264	166	203	118	56
0.58	846	616	509	352	423	335	210	257	149	71
0.57	1105	805	664	459	552	437	274	335	195	93
0.56	1503	1095	904	625	752	595	374	456	266	126

APPENDIX-A

TABLE A-3

Sample size required for testing population median (Continued)

P	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.55	2165	1577	1302	901	1082	856	538	657	383	182
0.54	3383	2464	2034	1407	1691	1338	841	1027	598	284
0.53	6013	4381	3616	2502	3006	2379	1494	1825	1063	506
0.52	13530	9857	8136	5628	6764	5353	3362	4106	2391	1137
0.51	54121	39428	32544	22514	27057	21411	13449	16425	9566	4550

''

APPENDIX-B

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

For the second sample $n_2 = n_1\{k/(1-k)\}$

k = 0.2

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	108	79	65	45	54	43	27	33	19	9
0.99	110	80	66	46	55	44	27	34	20	9
0.98	113	82	68	47	56	45	28	34	20	9
0.97	115	84	69	48	58	46	29	35	20	10
0.96	117	86	71	49	59	46	29	36	21	10
0.95	120	87	72	50	60	47	30	36	21	10
0.94	123	89	74	51	61	48	30	37	22	10
0.93	125	91	75	52	63	50	31	38	22	11
0.92	128	93	77	53	64	51	32	39	23	11
0.91	131	95	79	54	65	52	32	40	23	11
0.90	134	97	80	56	67	53	33	41	24	11
0.89	137	100	82	57	68	54	34	41	24	11
0.88	140	102	84	58	70	55	35	42	25	12
0.87	143	104	86	59	71	57	36	43	25	12
0.86	146	107	88	61	73	58	36	44	26	12
0.85	150	109	90	62	75	59	37	45	26	13
0.84	153	112	92	64	77	61	38	47	27	13
0.83	157	114	94	65	79	62	39	48	28	13
0.82	161	117	97	67	80	64	40	49	28	14
0.81	165	120	99	69	82	65	41	50	29	14
0.80	169	123	102	70	85	67	42	51	30	14
0.79	173	126	104	72	87	69	43	53	31	15
0.78	178	130	107	74	89	70	44	54	31	15
0.77	183	133	110	76	91	72	45	55	32	15
0.76	187	137	113	78	94	74	47	57	33	16
0.75	192	140	116	80	96	76	48	58	34	16
0.74	198	144	119	82	99	78	49	60	35	17
0.73	203	148	122	84	102	80	50	62	36	17
0.72	209	152	126	87	104	83	52	63	37	18
0.71	215	156	129	89	107	85	53	65	38	18
0.70	221	161	133	92	110	87	55	67	39	19
0.69	227	166	137	95	114	90	56	69	40	19
0.68	234	171	141	97	117	93	58	71	41	20
0.67	241	176	145	100	121	95	60	73	43	20
0.66	248	181	149	103	124	98	62	75	44	21
0.65	256	187	154	107	128	101	64	78	45	22
0.64	264	193	159	110	132	105	66	80	47	22
0.63	273	199	164	113	136	108	68	83	48	23
0.62	282	205	169	117	141	111	70	85	50	24
0.61	291	212	175	121	145	115	72	88	51	24
0.60	301	219	181	125	150	119	75	91	53	25

Note : $\Delta = |\mu^* - \mu_0^*| / \sigma$

APPENDIX-B

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.2 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.59	311	227	187	129	155	123	77	94	55	26
0.58	322	234	193	134	161	127	80	98	57	27
0.57	333	243	200	139	167	132	83	101	59	28
0.56	345	251	208	144	173	137	86	105	61	29
0.55	358	261	215	149	179	142	89	109	63	30
0.54	371	270	223	154	186	147	92	113	66	31
0.53	385	281	232	160	193	152	96	117	68	32
0.52	400	292	241	167	200	158	99	121	71	34
0.51	416	303	250	173	208	165	103	126	74	35
0.50	433	315	260	180	216	171	108	131	77	36
0.49	451	328	271	188	225	178	112	137	80	38
0.48	470	342	283	195	235	186	117	143	83	39
0.47	490	357	295	204	245	194	122	149	87	41
0.46	512	373	308	213	256	202	127	155	90	43
0.45	535	389	321	222	267	211	133	162	94	45
0.44	559	407	336	233	280	221	139	170	99	47
0.43	585	426	352	244	293	232	145	178	103	49
0.42	614	447	369	255	307	243	152	186	108	52
0.41	644	469	387	268	322	255	160	195	114	54
0.40	677	493	407	281	338	268	168	205	120	57
0.39	712	518	428	296	356	282	177	216	126	60
0.38	750	546	451	312	375	297	186	227	132	63
0.37	791	576	475	329	395	313	196	240	140	66
0.36	835	608	502	347	418	330	208	253	148	70
0.35	884	644	531	368	442	350	220	268	156	74
0.34	936	682	563	390	468	370	233	284	165	79
0.33	994	724	598	413	497	393	247	302	176	84
0.32	1057	770	636	440	528	418	263	321	187	89
0.31	1126	821	677	469	563	446	280	342	199	95
0.30	1203	876	723	500	601	476	299	365	213	101
0.29	1287	938	774	535	643	509	320	391	227	108
0.28	1381	1006	830	574	690	546	343	419	244	116
0.27	1485	1082	893	618	742	587	369	451	262	125
0.26	1601	1167	963	666	801	633	398	485	283	135
0.25	1732	1262	1041	720	866	685	430	526	306	146
0.24	1879	1369	1130	782	939	743	467	570	332	158
0.23	2046	1491	1230	851	1023	809	508	621	362	172
0.22	2236	1629	1345	930	1118	885	556	679	395	188
0.21	2454	1788	1476	1021	1227	971	610	745	434	206
0.20	2706	1971	1627	1126	1353	1071	672	821	478	227
0.19	2998	2184	1803	1247	1499	1186	745	910	530	252
0.18	3341	2434	2009	1390	1670	1322	830	1014	590	281
0.17	3745	2729	2252	1558	1872	1482	931	1137	662	315
0.16	4228	3080	2543	1759	2114	1673	1051	1283	747	355
0.15	4811	3505	2893	2001	2405	1903	1195	1460	850	404
0.14	5523	4023	3321	2297	2761	2185	1372	1676	976	464
0.13	6405	4666	3851	2664	3202	2534	1592	1944	1132	538
0.12	7517	5476	4520	3127	3758	2974	1868	2281	1329	632
0.11	8946	6517	5379	3721	4472	3539	2223	2715	1581	752
0.10	10824	7886	6509	4503	5411	4282	2690	3285	1913	910
0.09	13363	9735	8036	5559	6651	5287	3321	4056	2362	1123
0.08	16913	12321	10170	7035	8455	6691	4203	5133	2989	1422
0.07	22090	16093	13283	9189	11044	8739	5489	6704	3904	1857
0.06	30067	21904	18080	12508	15032	11895	7472	9125	5314	2528
0.05	43297	31542	26035	18011	21646	17129	10759	13140	7653	3640
0.04	67652	49285	40680	28142	33821	26764	16811	20531	11957	5687
0.03	120270	87618	72320	50030	60127	47580	29887	36500	21257	10110
0.02	270607	197140	162721	112568	135285	107055	67245	82125	47829	22748
0.01	1082427	788561	650883	450270	541139	428220	268981	328500	191316	90990

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.4 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	54	39	33	23	27	21	13	16	10	5
0.99	55	40	33	23	28	22	14	17	10	5
0.98	56	41	34	23	28	22	14	17	10	5
0.97	58	42	35	24	29	23	14	17	10	5
0.96	59	43	35	24	29	23	15	18	10	5
0.95	60	44	36	25	30	24	15	18	11	5
0.94	61	45	37	25	31	24	15	19	11	5
0.93	63	46	38	26	31	25	16	19	11	5
0.92	64	47	38	27	32	25	16	19	11	5
0.91	65	48	39	27	33	26	16	20	12	5
0.90	67	49	40	28	33	26	17	20	12	6
0.89	68	50	41	28	34	27	17	21	12	6
0.88	70	51	42	29	35	28	17	21	12	6
0.87	72	52	43	30	36	28	18	22	13	6
0.86	73	53	44	30	37	29	18	22	13	6
0.85	75	55	45	31	37	30	19	23	13	6
0.84	77	56	46	32	38	30	19	23	14	6
0.83	79	57	47	33	39	31	20	24	14	7
0.82	80	59	48	33	40	32	20	24	14	7
0.81	82	60	50	34	41	33	20	25	15	7
0.80	85	62	51	35	42	33	21	26	15	7
0.79	87	63	52	36	43	34	22	26	15	7
0.78	89	65	53	37	44	35	22	27	16	7
0.77	91	67	55	38	46	36	23	28	16	8
0.76	94	68	56	39	47	37	23	28	17	8
0.75	96	70	58	40	48	38	24	29	17	8
0.74	99	72	59	41	49	39	25	30	17	8
0.73	102	74	61	42	51	40	25	31	18	9
0.72	104	76	63	43	52	41	26	32	18	9
0.71	107	78	65	45	54	42	27	33	19	9
0.70	110	80	66	46	55	44	27	34	20	9
0.69	114	83	68	47	57	45	28	34	20	10
0.68	117	85	70	49	59	46	29	36	21	10
0.67	121	88	72	50	60	48	30	37	21	10
0.66	124	91	75	52	62	49	31	38	22	10
0.65	128	93	77	53	64	51	32	39	23	11
0.64	132	96	79	55	66	52	33	40	23	11
0.63	136	99	82	57	68	54	34	41	24	11
0.62	141	103	85	59	70	56	35	43	25	12
0.61	145	106	87	61	73	58	36	44	26	12
0.60	150	110	90	63	75	59	37	46	27	13
0.59	155	113	93	65	78	62	39	47	27	13
0.58	161	117	97	67	80	64	40	49	28	14
0.57	167	121	100	69	83	66	41	51	29	14
0.56	173	126	104	72	86	68	43	52	31	15
0.55	179	130	108	74	89	71	44	54	32	15
0.54	186	135	112	77	93	73	46	56	33	16
0.53	193	140	116	80	96	76	48	58	34	16
0.52	200	146	120	83	100	79	50	61	35	17
0.51	208	152	125	87	104	82	52	63	37	17
0.50	216	158	130	90	108	86	54	66	38	18
0.49	225	164	136	94	113	89	56	68	40	19
0.48	235	171	141	98	117	93	58	71	42	20
0.47	245	178	147	102	122	97	61	74	43	21
0.46	256	186	154	106	128	101	64	78	45	22
0.45	267	195	161	111	134	106	66	81	47	22
0.44	280	204	168	116	140	111	69	85	49	23
0.43	293	213	176	122	146	116	73	89	52	25
0.42	307	224	184	128	153	121	76	93	54	26
0.41	322	235	194	134	161	127	80	98	57	27
0.40	338	246	203	141	169	134	84	103	60	28
0.39	356	259	214	148	178	141	88	108	63	30
0.38	375	273	225	156	187	148	93	114	66	32
0.37	395	288	238	164	198	156	98	120	70	33
0.36	418	304	251	174	209	165	104	127	74	35

APPENDIX-B TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.4 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.35	442	322	266	184	221	175	110	134	78	37
0.34	468	341	282	195	234	185	116	142	83	39
0.33	497	362	299	207	248	197	123	151	88	42
0.32	529	385	318	220	264	209	131	160	93	44
0.31	563	410	339	234	282	223	140	171	100	47
0.30	601	438	362	250	301	238	149	182	106	51
0.29	644	469	387	268	322	255	160	195	114	54
0.28	690	503	415	287	345	273	172	210	122	58
0.27	742	541	446	309	371	294	184	225	131	62
0.26	801	583	481	333	400	317	199	243	142	67
0.25	866	631	521	360	433	343	215	263	153	73
0.24	940	685	565	391	470	372	233	285	166	79
0.23	1023	745	615	426	511	405	254	310	181	86
0.22	1118	815	672	465	559	442	278	339	198	94
0.21	1227	894	738	511	614	486	305	372	217	103
0.20	1353	986	814	563	676	535	336	411	239	114
0.19	1499	1092	902	624	750	593	373	455	265	126
0.18	1670	1217	1004	695	835	661	415	507	295	140
0.17	1873	1364	1126	779	936	741	465	568	331	157
0.16	2114	1540	1271	879	1057	836	525	642	374	178
0.15	2405	1752	1446	1001	1203	952	598	730	425	202
0.14	2761	2012	1660	1149	1380	1092	686	838	488	232
0.13	3202	2333	1926	1332	1601	1267	796	972	566	269
0.12	3758	2738	2260	1563	1879	1487	934	1141	664	316
0.11	4473	3259	2690	1861	2236	1770	1111	1357	791	376
0.10	5412	3943	3254	2251	2706	2141	1345	1642	957	455
0.09	6682	4868	4018	2779	3340	2643	1660	2028	1181	562
0.08	8456	6161	5085	3518	4228	3345	2101	2566	1495	711
0.07	11045	8047	6642	4595	5522	4370	2745	3352	1952	928
0.06	15034	10952	9040	6254	7516	5948	3736	4562	2657	1264
0.05	21649	15771	13018	9005	10823	8564	5380	6570	3826	1820
0.04	33826	24643	20340	14071	16911	13382	8406	10266	5979	2843
0.03	60135	43809	36160	25015	30063	23790	14943	18250	10629	5055
0.02	135303	98570	81360	56284	67642	53528	33623	41062	23915	11374
0.01	541214	394281	325442	225135	270570	214110	134490	164250	95658	45495

APPENDIX-B TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.6 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	36	26	22	15	18	14	9	11	6	3
0.99	37	27	22	15	18	15	9	11	7	3
0.98	38	27	23	16	19	15	9	11	7	3
0.97	38	28	23	16	19	15	10	12	7	3
0.96	39	29	24	16	20	15	10	12	7	3
0.95	40	29	24	17	20	16	10	12	7	3
0.94	41	30	25	17	20	16	10	12	7	3
0.93	42	30	25	17	21	17	10	13	7	4
0.92	43	31	26	18	21	17	11	13	8	4
0.91	44	32	26	18	22	17	11	13	8	4
0.90	45	32	27	19	22	18	11	14	8	4
0.89	46	33	27	19	23	18	11	14	8	4
0.88	47	34	28	19	23	18	12	14	8	4
0.87	48	35	29	20	24	19	12	14	8	4
0.86	49	36	29	20	24	19	12	15	9	4
0.85	50	36	30	21	25	20	12	15	9	4

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.6 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.84	51	37	31	21	26	20	13	16	9	4
0.83	52	38	31	22	26	21	13	16	9	4
0.82	54	39	32	22	27	21	13	16	9	5
0.81	55	40	33	23	27	22	14	17	10	5
0.80	56	41	34	23	28	22	14	17	10	5
0.79	58	42	35	24	29	23	14	18	10	5
0.78	59	43	36	25	30	23	15	18	10	5
0.77	61	44	37	25	30	24	15	18	11	5
0.76	62	46	38	26	31	25	16	19	11	5
0.75	64	47	39	27	32	25	16	19	11	5
0.74	66	48	40	27	33	26	16	20	12	6
0.73	68	49	41	28	34	27	17	21	12	6
0.72	70	51	42	29	35	28	17	21	12	6
0.71	72	52	43	30	36	28	18	22	13	6
0.70	74	54	44	31	37	29	18	22	13	6
0.69	76	55	46	32	38	30	19	23	13	6
0.68	78	57	47	32	39	31	19	24	14	7
0.67	80	59	48	33	40	32	20	24	14	7
0.66	83	60	50	34	41	33	21	25	15	7
0.65	85	62	51	36	43	34	21	26	15	7
0.64	88	64	53	37	44	35	22	27	16	7
0.63	91	66	55	38	45	36	23	28	16	8
0.62	94	68	56	39	47	37	23	28	17	8
0.61	97	71	58	40	48	38	24	29	17	8
0.60	100	73	60	42	50	40	25	30	18	8
0.59	104	76	62	43	52	41	26	31	18	9
0.58	107	78	64	45	54	42	27	33	19	9
0.57	111	81	67	46	56	44	28	34	20	9
0.56	115	84	69	48	58	46	29	35	20	10
0.55	119	87	72	50	60	47	30	36	21	10
0.54	124	90	74	51	62	49	31	38	22	10
0.53	128	94	77	53	64	51	32	39	23	11
0.52	133	97	80	56	67	53	33	40	24	11
0.51	139	101	83	58	69	55	34	42	25	12
0.50	144	105	87	60	72	57	36	44	26	12
0.49	150	109	90	63	75	59	37	46	27	13
0.48	157	114	94	65	78	62	39	48	28	13
0.47	163	119	98	68	82	65	41	50	29	14
0.46	171	124	103	71	85	67	42	52	30	14
0.45	178	130	107	74	89	70	44	54	31	15
0.44	186	136	112	78	93	74	46	57	33	16
0.43	195	142	117	81	98	77	48	59	34	16
0.42	205	149	123	85	102	81	51	62	36	17
0.41	215	156	129	89	107	85	53	65	38	18
0.40	226	164	136	94	113	89	56	68	40	19
0.39	237	173	143	99	119	94	59	72	42	20
0.38	250	182	150	104	125	99	62	76	44	21
0.37	264	192	158	110	132	104	65	80	47	22
0.36	278	203	167	116	139	110	69	84	49	23
0.35	295	215	177	123	147	117	73	89	52	25
0.34	312	227	188	130	156	123	78	95	55	26
0.33	331	241	199	138	166	131	82	101	59	28
0.32	352	257	212	147	176	139	88	107	62	30
0.31	375	274	226	156	188	149	93	114	66	32
0.30	401	292	241	167	200	159	100	122	71	34
0.29	429	313	258	178	214	170	107	130	76	36
0.28	460	335	277	191	230	182	114	140	81	39
0.27	495	361	298	206	247	196	123	150	87	42
0.26	534	389	321	222	267	211	133	162	94	45
0.25	577	421	347	240	289	228	143	175	102	49
0.24	626	456	377	261	313	248	156	190	111	53
0.23	682	497	410	284	341	270	169	207	121	57
0.22	745	543	448	310	373	295	185	226	132	63
0.21	818	596	492	340	409	324	203	248	145	69
0.20	902	657	542	375	451	357	224	274	159	76

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.6 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.19	999	728	601	416	500	395	248	303	177	84
0.18	1114	811	670	463	557	441	277	338	197	94
0.17	1248	910	751	519	624	494	310	379	221	105
0.16	1409	1027	848	586	705	558	350	428	249	118
0.15	1604	1168	964	667	802	634	398	487	283	135
0.14	1841	1341	1107	766	920	728	457	559	325	155
0.13	2135	1555	1284	888	1067	845	531	648	377	179
0.12	2506	1825	1507	1042	1253	991	623	760	443	211
0.11	2982	2172	1793	1240	1491	1180	741	905	527	251
0.10	3608	2629	2170	1501	1804	1427	897	1095	638	303
0.09	4454	3245	2679	1853	2227	1762	1107	1352	787	374
0.08	5638	4107	3390	2345	2818	2230	1401	1711	996	474
0.07	7363	5364	4428	3063	3681	2913	1830	2235	1301	619
0.06	10022	7301	6027	4169	5011	3965	2491	3042	1771	843
0.05	14432	10514	8678	6004	7215	5710	3586	4380	2551	1213
0.04	22551	16428	13560	9381	11274	8921	5604	6844	3986	1896
0.03	40090	29206	24107	16677	20042	15860	9962	12167	7086	3370
0.02	90202	65713	54240	37523	45095	35685	22415	27375	15943	7583
0.01	360809	262854	216961	150090	180380	142740	89660	109500	63772	30330

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.8 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	27	20	16	11	14	11	7	8	5	2
0.99	28	20	17	11	14	11	7	8	5	2
0.98	28	21	17	12	14	11	7	9	5	2
0.97	29	21	17	12	14	11	7	9	5	2
0.96	29	21	18	12	15	12	7	9	5	2
0.95	30	22	18	12	15	12	7	9	5	3
0.94	31	22	18	13	15	12	8	9	5	3
0.93	31	23	19	13	16	12	8	9	6	3
0.92	32	23	19	13	16	13	8	10	6	3
0.91	33	24	20	14	16	13	8	10	6	3
0.90	33	24	20	14	17	13	8	10	6	3
0.89	34	25	21	14	17	14	8	10	6	3
0.88	35	25	21	15	17	14	9	11	6	3
0.87	36	26	21	15	18	14	9	11	6	3
0.86	37	27	22	15	18	14	9	11	6	3
0.85	37	27	23	16	19	15	9	11	7	3
0.84	38	28	23	16	19	15	10	12	7	3
0.83	39	29	24	16	20	16	10	12	7	3
0.82	40	29	24	17	20	16	10	12	7	3
0.81	41	30	25	17	21	16	10	13	7	3
0.80	42	31	25	18	21	17	11	13	7	4
0.79	43	32	26	18	22	17	11	13	8	4
0.78	44	32	27	19	22	18	11	13	8	4
0.77	46	33	27	19	23	18	11	14	8	4
0.76	47	34	28	19	23	19	12	14	8	4
0.75	48	35	29	20	24	19	12	15	9	4
0.74	49	36	30	21	25	20	12	15	9	4
0.73	51	37	31	21	25	20	13	15	9	4
0.72	52	38	31	22	26	21	13	16	9	4
0.71	54	39	32	22	27	21	13	16	9	5
0.70	55	40	33	23	28	22	14	17	10	5
0.69	57	41	34	24	28	22	14	17	10	5

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.8 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.68	59	43	35	24	29	23	15	18	10	5
0.67	60	44	36	25	30	24	15	18	11	5
0.66	62	45	37	26	31	25	15	19	11	5
0.65	64	47	39	27	32	25	16	19	11	5
0.64	66	48	40	27	33	26	16	20	12	6
0.63	68	50	41	28	34	27	17	21	12	6
0.62	70	51	42	29	35	28	17	21	12	6
0.61	73	53	44	30	36	29	18	22	13	6
0.60	75	55	45	31	38	30	19	23	13	6
0.59	78	57	47	32	39	31	19	24	14	7
0.58	80	59	48	33	40	32	20	24	14	7
0.57	83	61	50	35	42	33	21	25	15	7
0.56	86	63	52	36	43	34	21	26	15	7
0.55	89	65	54	37	45	35	22	27	16	8
0.54	93	68	56	39	46	37	23	28	16	8
0.53	96	70	58	40	48	38	24	29	17	8
0.52	100	73	60	42	50	40	25	30	18	8
0.51	104	76	63	43	52	41	26	32	18	9
0.50	108	79	65	45	54	43	27	33	19	9
0.49	113	82	68	47	56	45	28	34	20	9
0.48	117	86	71	49	59	46	29	36	21	10
0.47	123	89	74	51	61	48	30	37	22	10
0.46	128	93	77	53	64	51	32	39	23	11
0.45	134	97	80	56	67	53	33	41	24	11
0.44	140	102	84	58	70	55	35	42	25	12
0.43	146	107	88	61	73	58	36	44	26	12
0.42	153	112	92	64	77	61	38	47	27	13
0.41	161	117	97	67	80	64	40	49	28	14
0.40	169	123	102	70	85	67	42	51	30	14
0.39	178	130	107	74	89	70	44	54	31	15
0.38	187	137	113	78	94	74	47	57	33	16
0.37	198	144	119	82	99	78	49	60	35	17
0.36	209	152	126	87	104	83	52	63	37	18
0.35	221	161	133	92	110	87	55	67	39	19
0.34	234	171	141	97	117	93	58	71	41	20
0.33	248	181	149	103	124	98	62	75	44	21
0.32	264	193	159	110	132	105	66	80	47	22
0.31	282	205	169	117	141	111	70	85	50	24
0.30	301	219	181	125	150	119	75	91	53	25
0.29	322	234	193	134	161	127	80	98	57	27
0.28	345	251	208	144	173	137	86	105	61	29
0.27	371	270	223	154	186	147	92	113	66	31
0.26	400	292	241	167	200	158	99	121	71	34
0.25	433	315	260	180	216	171	108	131	77	36
0.24	470	342	283	195	235	186	117	143	83	39
0.23	512	373	308	213	256	202	127	155	90	43
0.22	559	407	336	233	280	221	139	170	99	47
0.21	614	447	369	255	307	243	152	186	108	52
0.20	677	493	407	281	338	268	168	205	120	57
0.19	750	546	451	312	375	297	186	227	132	63
0.18	835	608	502	347	418	330	208	253	148	70
0.17	936	682	563	390	468	370	233	284	165	79
0.16	1057	770	636	440	528	418	263	321	187	89
0.15	1203	876	723	500	601	476	299	365	213	101
0.14	1381	1006	830	574	690	546	343	419	244	116
0.13	1601	1167	963	666	801	633	398	486	283	135
0.12	1879	1369	1130	782	939	743	467	570	332	158
0.11	2236	1629	1345	930	1118	885	556	679	395	188
0.10	2706	1971	1627	1126	1353	1071	672	821	478	227
0.09	3341	2434	2009	1390	1670	1322	830	1014	590	281
0.08	4228	3080	2543	1759	2114	1673	1051	1283	747	355
0.07	5523	4023	3321	2297	2761	2185	1372	1676	976	464
0.06	7517	5476	4520	3127	3758	2974	1868	2281	1329	632
0.05	10824	7886	6509	4503	5411	4282	2690	3285	1913	910
0.04	16913	12321	10170	7035	8455	6691	4203	5133	2989	1422

APPENDIX-B

TABLE B-1

Sample size required for testing the difference between two means

Values for the first sample n_1

k = 0.8 *(Continued)*

Δ	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.03	30067	21904	18080	12508	15032	11895	7472	9125	5314	2528
0.02	67652	49285	40680	28142	33821	26764	16811	20531	11957	5687
0.01	270607	197140	162721	112568	135285	107055	67245	82125	47829	22748

APPENDIX-B

TABLE B-2

Sample size required for testing the difference between two population distributions

Values for the first sample n

k = 0.2 For the second sample m = n {k/(1-k)}

p	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	36	26	22	15	18	14	9	11	6	3
0.99	38	27	23	16	19	15	9	11	7	3
0.98	39	29	24	16	20	15	10	12	7	3
0.97	41	30	25	17	20	16	10	12	7	3
0.96	43	31	26	18	21	17	11	13	8	4
0.95	45	32	27	19	22	18	11	14	8	4
0.94	47	34	28	19	23	18	12	14	8	4
0.93	49	36	29	20	24	19	12	15	9	4
0.92	51	37	31	21	26	20	13	16	9	4
0.91	54	39	32	22	27	21	13	16	9	5
0.90	56	41	34	23	28	22	14	17	10	5
0.89	59	43	36	25	30	23	15	18	10	5
0.88	62	46	38	26	31	25	16	19	11	5
0.87	66	48	40	27	33	26	16	20	12	6
0.86	70	51	42	29	35	28	17	21	12	6
0.85	74	54	44	31	37	29	18	22	13	6
0.84	78	57	47	32	39	31	19	24	14	7
0.83	83	60	50	34	41	33	21	25	15	7
0.82	88	64	53	37	44	35	22	27	16	7
0.81	94	68	56	39	47	37	23	28	17	8
0.80	100	73	60	42	50	40	25	30	18	8
0.79	107	78	64	45	54	42	27	33	19	9
0.78	115	84	69	48	58	46	29	35	20	10
0.77	124	90	74	51	62	49	31	38	22	10
0.76	133	97	80	56	67	53	33	40	24	11
0.75	144	105	87	60	72	57	36	44	26	12
0.74	157	114	94	65	78	62	39	48	28	13
0.73	171	124	103	71	85	67	42	52	30	14
0.72	186	136	112	78	93	74	46	57	33	16
0.71	205	149	123	85	102	81	51	62	36	17
0.70	226	164	135	94	113	89	56	68	40	19
0.69	250	182	150	104	125	99	62	76	44	21
0.68	278	203	167	116	139	110	69	84	49	23
0.67	312	227	188	130	156	123	78	95	55	26
0.66	352	257	212	147	176	139	88	107	62	30
0.65	401	292	241	167	200	159	100	122	71	34
0.64	460	335	277	191	230	182	114	140	81	39
0.63	534	389	321	222	267	211	133	162	94	45
0.62	626	456	377	261	313	248	156	190	111	53
0.61	745	543	448	310	373	295	185	226	132	63
0.60	902	657	542	375	451	357	224	274	159	76
0.59	1114	811	670	463	557	441	277	338	197	94
0.58	1409	1027	848	586	705	558	350	428	249	118
0.57	1841	1341	1107	766	920	728	457	559	325	155
0.56	2506	1825	1507	1042	1253	991	623	760	443	211
0.55	3608	2629	2170	1501	1804	1427	897	1095	638	303
0.54	5638	4107	3390	2345	2818	2230	1401	1711	996	474
0.53	10022	7301	6027	4169	5011	3965	2491	3042	1771	843

TABLE B-2

Sample size required for testing the difference between two population distributions

Values for the first sample n

k = 0.2 *(Continued)*

p	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
0.52	22551	16428	13560	9381	11274	8921	5604	6844	3986	1896
0.51	90202	65713	54240	37523	45095	35685	22415	27375	15943	7583

TABLE B-2

Sample size required for testing the difference between two population distributions

Values for the first sample n

k = 0.4 *(Continued)*

p	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	18	13	11	8	9	7	4	5	3	2
0.99	19	14	11	8	9	7	5	6	3	2
0.98	20	14	12	8	10	8	5	6	3	2
0.97	20	15	12	8	10	8	5	6	4	2
0.96	21	16	13	9	11	8	5	6	4	2
0.95	22	16	13	9	11	9	6	7	4	2
0.94	23	17	14	10	12	9	6	7	4	2
0.93	24	18	15	10	12	10	6	7	4	2
0.92	26	19	15	11	13	10	6	8	5	2
0.91	27	20	16	11	13	11	7	8	5	2
0.90	28	21	17	12	14	11	7	9	5	2
0.89	30	22	18	12	15	12	7	9	5	2
0.88	31	23	19	13	16	12	8	9	6	3
0.87	33	24	20	14	16	13	8	10	6	3
0.86	35	25	21	14	17	14	9	11	6	3
0.85	37	27	22	15	18	15	9	11	7	3
0.84	39	28	23	16	20	15	10	12	7	3
0.83	41	30	25	17	21	16	10	13	7	3
0.82	44	32	26	18	22	17	11	13	8	4
0.81	47	34	28	20	23	19	12	14	8	4
0.80	50	37	30	21	25	20	12	15	9	4
0.79	54	39	32	22	27	21	13	16	9	5
0.78	58	42	35	24	29	23	14	17	10	5
0.77	62	45	37	26	31	24	15	19	11	5
0.76	67	49	40	28	33	26	17	20	12	6
0.75	72	53	43	30	36	29	18	22	13	6
0.74	78	57	47	33	39	31	19	24	14	7
0.73	85	62	51	35	43	34	21	26	15	7
0.72	93	68	56	39	47	37	23	28	16	8
0.71	102	75	61	43	51	40	25	31	18	9
0.70	113	82	68	47	56	45	28	34	20	9
0.69	125	91	75	52	62	49	31	38	22	11
0.68	139	101	84	58	70	55	35	42	25	12
0.67	156	114	94	65	78	62	39	47	28	13
0.66	176	128	106	73	83	70	44	53	31	15
0.65	200	146	121	83	100	79	50	61	35	17
0.64	230	168	138	96	115	91	57	70	41	19
0.63	267	194	160	111	133	106	66	81	47	22
0.62	313	228	188	130	157	124	78	95	55	26
0.61	373	272	224	155	186	147	93	113	66	31
0.60	451	329	271	188	225	178	112	137	80	38
0.59	557	406	335	232	278	220	138	169	98	47
0.58	705	513	424	293	352	279	175	214	125	59
0.57	920	671	553	383	460	364	229	279	163	77
0.56	1253	913	753	521	626	496	311	380	221	105
0.55	1804	1314	1085	750	902	714	448	547	319	152
0.54	2819	2054	1695	1173	1409	1115	700	855	498	237
0.53	5011	3651	3013	2085	2505	1983	1245	1521	886	421
0.52	11275	8214	6780	4690	5637	4461	2802	3422	1993	948
0.51	45101	32857	27120	18761	22547	17843	11208	13687	7972	3791

TABLE B-2

Sample size required for testing the difference between two population distributions

Values for the first sample n

k = 0.6 *(Continued)*

p	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	12	9	7	5	6	5	3	4	2	1
0.99	13	9	8	5	6	5	3	4	2	1
0.98	13	10	8	5	7	5	3	4	2	1
0.97	14	10	8	6	7	5	3	4	2	1
0.96	14	10	9	6	7	6	4	4	3	1
0.95	15	11	9	6	7	6	4	5	3	1
0.94	16	11	9	6	8	6	4	5	3	1
0.93	16	12	10	7	8	6	4	5	3	1
0.92	17	12	10	7	9	7	4	5	3	1
0.91	18	13	11	7	9	7	4	5	3	2
0.90	19	14	11	8	9	7	5	6	3	2
0.89	20	14	12	8	10	8	5	6	3	2
0.88	21	15	13	9	10	8	5	6	4	2
0.87	22	16	13	9	11	9	5	7	4	2
0.86	23	17	14	10	12	9	6	7	4	2
0.85	25	18	15	10	12	10	6	7	4	2
0.84	26	19	16	11	13	10	6	8	5	2
0.83	28	20	17	11	14	11	7	8	5	2
0.82	29	21	18	12	15	12	7	9	5	2
0.81	31	23	19	13	16	12	8	9	6	3
0.80	33	24	20	14	17	13	8	10	6	3
0.79	36	26	21	15	18	14	9	11	6	3
0.78	38	28	23	16	19	15	10	12	7	3
0.77	41	30	25	17	21	16	10	13	7	3
0.76	44	32	27	19	22	18	11	13	8	4
0.75	48	35	29	20	24	19	12	15	9	4
0.74	52	38	31	22	26	21	13	16	9	4
0.73	57	41	34	24	28	22	14	17	10	5
0.72	62	45	37	26	31	25	15	19	11	5
0.71	68	50	41	28	34	27	17	21	12	6
0.70	75	55	45	31	38	30	19	23	13	6
0.69	83	61	50	35	42	33	21	25	15	7
0.68	93	68	56	39	46	37	23	28	16	8
0.67	104	76	63	43	52	41	26	32	18	9
0.66	117	86	71	49	59	46	29	36	21	10
0.65	134	97	80	56	67	53	33	41	24	11
0.64	153	112	92	64	77	61	38	47	27	13
0.63	178	130	107	74	89	70	44	54	31	15
0.62	209	152	126	87	104	83	52	63	37	18
0.61	248	181	149	103	124	98	62	75	44	21
0.60	301	219	181	125	150	119	75	91	53	25
0.59	371	270	223	154	186	147	92	113	66	31
0.58	470	342	263	195	235	186	117	143	83	39
0.57	614	447	369	255	307	243	152	186	108	52
0.56	835	608	502	347	418	330	208	253	148	70
0.55	1203	876	723	500	601	476	299	365	213	101
0.54	1879	1369	1130	782	939	743	467	570	332	158
0.53	3341	2434	2009	1390	1670	1322	830	1014	590	281
0.52	7517	5476	4520	3127	3758	2974	1868	2281	1329	632
0.51	30067	21904	18080	12503	15032	11895	7472	9125	5314	2528

APPENDIX-B TABLE B-2

Sample size required for testing the difference between two population distributions

Values for the first sample n

k = 0.8 *(Continued)*

p	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	9	7	5	4	5	4	2	3	2	1
0.99	9	7	6	4	5	4	2	3	2	1
0.98	10	7	6	4	5	4	2	3	2	1
0.97	10	7	6	4	5	4	3	3	2	1
0.96	11	8	6	4	5	4	3	3	2	1
0.95	11	8	7	5	6	4	3	3	2	1
0.94	12	8	7	5	6	5	3	4	2	1
0.93	12	9	7	5	6	5	3	4	2	1
0.92	13	9	8	5	6	5	3	4	2	1
0.91	13	10	8	6	7	5	3	4	2	1
0.90	14	10	8	6	7	6	4	4	2	1
0.89	15	11	9	6	7	6	4	4	3	1
0.88	16	11	9	6	8	6	4	5	3	1
0.87	16	12	10	7	8	7	4	5	3	1
0.86	17	13	10	7	9	7	4	5	3	1
0.85	18	13	11	8	9	7	5	6	3	2
0.84	20	14	12	8	10	8	5	6	3	2
0.83	21	15	12	9	10	8	5	6	4	2
0.82	22	16	13	9	11	9	5	7	4	2
0.81	23	17	14	10	12	9	6	7	4	2
0.80	25	18	15	10	13	10	6	8	4	2
0.79	27	20	16	11	13	11	7	8	5	2
0.78	29	21	17	12	14	11	7	9	5	2
0.77	31	23	19	13	15	12	8	9	5	3
0.76	33	24	20	14	17	13	8	10	6	3
0.75	36	26	22	15	18	14	9	11	6	3
0.74	39	29	24	16	20	15	10	12	7	3
0.73	43	31	26	18	21	17	11	13	8	4
0.72	47	34	28	19	23	18	12	14	8	4
0.71	51	37	31	21	26	20	13	16	9	4
0.70	56	41	34	23	28	22	14	17	10	5
0.69	62	46	38	26	31	25	16	19	11	5
0.68	70	51	42	29	35	28	17	21	12	6
0.67	78	57	47	32	39	31	19	24	14	7
0.66	83	64	53	37	44	35	22	27	16	7
0.65	100	73	60	42	50	40	25	30	18	8
0.64	115	84	69	48	58	46	29	35	20	10
0.63	133	97	80	56	67	53	33	40	24	11
0.62	157	114	94	65	78	62	39	48	28	13
0.61	186	136	112	78	93	74	46	57	33	16
0.60	226	164	136	94	113	89	56	68	40	19
0.59	278	203	167	116	139	110	69	84	49	23
0.58	352	257	212	147	176	139	88	107	62	30
0.57	460	335	277	191	230	182	114	140	81	39
0.56	626	456	377	261	313	248	156	190	111	53
0.55	902	657	542	375	451	357	224	274	159	76
0.54	1409	1027	848	586	705	558	350	428	249	118
0.53	2506	1825	1507	1042	1253	991	623	760	443	211
0.52	5638	4107	3390	2345	2818	2230	1401	1711	996	474
0.51	22551	16428	13560	9381	11274	8921	5604	6844	3986	1896

APPENDIX-B

TABLE B-3

Sample size required for testing the significance of correlation coefficient

Δ_z	α=1% β=1%	α=1% β=5%	α=1% β=10%	α=1% β=25%	α=5% β=5%	α=5% β=10%	α=5% β=25%	α=10% β=10%	α=10% β=25%	α=25% β=25%
1.00	25	19	16	12	14	12	8	10	7	5
0.90	30	22	19	14	16	14	10	11	8	5
0.80	37	28	23	17	20	16	11	13	9	6
0.70	47	35	30	21	25	20	14	16	11	7
0.60	63	47	39	28	33	27	18	21	14	8
0.50	90	66	55	39	46	37	25	29	18	10
0.40	138	102	84	59	71	57	37	44	27	14
0.30	244	178	148	103	123	98	63	76	46	23
0.20	544	397	328	228	274	217	137	167	99	48
0.19	603	440	364	252	303	240	152	185	109	240
0.18	671	490	405	281	337	267	169	206	121	267
0.17	752	549	453	315	377	299	189	230	135	299
0.16	849	619	512	355	426	338	213	260	152	338
0.15	965	704	582	403	484	384	242	295	173	384
0.14	1108	808	667	462	555	440	277	338	198	440
0.13	1284	936	773	536	643	510	321	392	229	510
0.12	1506	1098	907	628	755	598	377	459	269	598
0.11	1792	1306	1079	747	897	711	448	546	319	711
0.10	2168	1580	1305	904	1085	859	541	660	386	859
0.09	2676	1950	1610	1115	1339	1060	667	814	475	1060
0.08	3386	2467	2037	1410	1694	1341	844	1030	601	1341
0.07	4421	3222	2660	1841	2212	1751	1101	1344	784	1751
0.06	6016	4384	3619	2505	3009	2382	1497	1828	1066	2382
0.05	8662	6311	5210	3605	4332	3429	2155	2631	1534	3429
0.04	13533	9860	8139	5631	6767	5356	3365	4109	2394	5356
0.03	24057	17527	14467	10009	12028	9519	5980	7303	4254	9519
0.02	54124	39431	32547	22517	27060	21414	13452	16428	9569	21414
0.01	216488	157715	130180	90057	108231	85647	53799	65703	38266	85647

Note : $\Delta_z = | z_p - z_{p0} |$